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2002 J. Phys.: Condens. Matter 14 7933

(http://iopscience.iop.org/0953-8984/14/34/311)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) 7933–7939 PII: S0953-8984(02)37711-7

Weak-localization and rectification current in non-diffusive quantum wires

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Received 5 June 2002
Published 15 August 2002
Online at stacks.iop.org/JPhysCM/14/7933

Abstract

We show that electron transport in disordered quantum wires can be described by a modified Cooperon equation, which coincides in form with the Dirac equation for the massive fermions in a (1 + 1)-dimensional system. In this new formalism, we calculate the direct electric current induced by electromagnetic (EM) fields in quasi-one-dimensional rings. This current changes sign, from diamagnetic to paramagnetic, depending on the amplitude and frequency of the time-dependent external EM field.

Impurity scattering and quantum coherence of the electron wavefunction are the two key concepts in the transport phenomena of mesoscopic conductors. Conduction electrons are weakly localized by the coherent back-scattering due to impurities, resulting in the effect commonly known as 'weak localization'. The conventional theory of weak localization [1] assumes deeply diffusive systems—i.e., the electron mean free path l is much shorter than the system size. Recently, experimental studies of transport phenomena in non-diffusive systems have also become important, primarily due to the recent progress in fabrication of clean nanostructures. In this paper, we present a formalism of the weak-localization phenomena, valid also in non-diffusive regimes. In particular, we consider electromagnetic (EM) field-induced current in mesoscopic rings, which is currently an important issue as regards the sign of the measured persistent current [2–4].

Nonlinear properties of field-induced current in mesoscopic rings have been studied in great detail for the case of the deeply diffusive regime [5, 6]. This problem has recently gained attention again due to its relevance to the problem of anomalously large persistent current [2–4] and low-temperature saturation of decoherence time [7–9]. We investigate the same physical model without using the diffusion approximation, which is valid only for $l \ll L$. The particular system considered in this paper is a quantum wire in the ring geometry with finite width much larger than the Fermi wavelength but smaller than the phase coherence length. We show that rectified direct currents in mesoscopic rings induced by high-frequency magnetic fields have oscillating sign depending on the frequency. This result sheds some light on the recent puzzle of the measured sign of the induced DC in mesoscopic quantum rings [3, 4, 9].

We start with the conventional weak-localization theory. Central to quantum transport in disordered conductors is the concept of the so-called 'Cooperon', the particle–particle diffusion propagator [1, 10]. The Cooperon is a two-particle Green function averaged over disorder configurations. In the presence of an EM field $\boldsymbol{A}$, the Cooperon is the retarded classical propagator of a modified diffusion equation:

$$\left[\frac{\partial}{\partial t} - D\left(\nabla_r - \frac{2\mathrm{i}e}{\hbar c}\boldsymbol{A}\right)^2 + \frac{1}{\tau_\phi}\right]C(\boldsymbol{r}, t; \boldsymbol{r}', t') = \delta(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t'), \tag{1}$$

where $D = v_F l/d$ is the diffusion coefficient for the d-dimensional system, v_F is the Fermi velocity, and τ_ϕ is the phase coherence time. This expression in equation (1) has proven to be useful in many cases, since one can easily consider geometrical effects through the boundary condition of the equation. However, it is worthwhile to note that equation (1) is only valid in the deeply diffusive regime. Even its original derivation hinged on the realization that the Fourier transformed Cooperon $C(\boldsymbol{Q}, \omega)$ could be approximated as [1, 11] $C(\boldsymbol{Q}, \omega) \approx 1/(-\mathrm{i}\omega + DQ^2)$ when

$$Ql \ll 1 \qquad \text{and} \qquad \omega\tau \ll 1, \tag{2}$$

where $\tau = l/v_F$ is the elastic mean free time.

When ω is not too small compared to $1/\tau$, one relies on the semiclassical Boltzmann theory [12] instead of equation (1) for the Cooperon; the semiclassical Boltzmann theory is free from the above constraining approximation in equation (2). In this theory [12], the electron motion is characterized by a function $F(\boldsymbol{r}, \boldsymbol{v}, t; \boldsymbol{r}', \boldsymbol{v}', t')$ which is a conditional probability density describing the probability of the particle initially at position $\boldsymbol{r}'$ at time t' with velocity $\boldsymbol{v}'$ being found at the position $\boldsymbol{r}$ at time t with the velocity $\boldsymbol{v}$. The intrinsic velocity of the particle is fixed as the Fermi velocity, $|\boldsymbol{v}| = |\boldsymbol{v}'| = v_F$. The conditional probability density F is the propagator of the distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ which satisfies the Boltzmann equation

$$\left[\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\left(\nabla_r - \frac{2\mathrm{i}e}{\hbar c}\boldsymbol{A}\right)\right]f = -\frac{f - f_0}{\tau} - \frac{f}{\tau_\phi} \tag{3}$$

$$f_0(\boldsymbol{r}, t) = \frac{1}{\mathcal{N}}\sum_{\boldsymbol{v}} f(\boldsymbol{r}, \boldsymbol{v}, t) \tag{4}$$

where $\mathcal{N}$ is the number of available values of $\boldsymbol{v}$.

From this point onwards, we will consider a mesoscopic quantum wire with finite width W in two dimensions. The boundary condition is that $f(\boldsymbol{r}, \boldsymbol{v}, t)$ is zero if $v_y \neq 0$ at the boundary of the wire $\boldsymbol{r} = x\hat{\boldsymbol{x}} \pm (W/2)\hat{\boldsymbol{y}}$. This condition is due to the fact that the electron number is conserved in the electron scattering with the boundary. We impose another condition: that the width of the wire W is small enough that $W/\tau_\phi \ll v_F$. In this case, the main contribution to the electron propagator F is essentially the zero mode in the transverse direction, i.e., there is no y-dependence in F. To be consistent with the boundary condition, $v_y = 0$ at the boundaries, we have only two values of $\boldsymbol{v}$ in the longitudinal direction, either $v_F\hat{\boldsymbol{x}}$ or $-v_F\hat{\boldsymbol{x}}$. The equation of the propagator for the Boltzmann equation (3) can be rewritten as a differential equation in a 2-by-2 matrix form:

$$\left[\frac{\partial}{\partial t} + v_F\sigma_z\left(\frac{\partial}{\partial x} - \frac{2\mathrm{i}e}{\hbar c}A\right) + \frac{1}{2\tau}(1 - \sigma_x) + \frac{1}{\tau_\phi}\right]\mathsf{F}(x, t; x', t') = \delta(x - x')\delta(t - t'), \tag{5}$$

where

$$\mathsf{F}(x, t; x', t') = \begin{pmatrix} F(x, v_F, t; x', v_F, t') & F(x, v_F, t; x', -v_F, t') \\ F(x, -v_F, t; x', v_F, t') & F(x, -v_F, t; x', -v_F, t') \end{pmatrix}. \tag{6}$$

As is clear in equation (6), each component of the above matrix formalism denotes the relevant chirality of the moving particle. Since the Cooperon $C(x, t; x', t')$ is also a probability density giving the probability for the particle initially at (x', t') to be found at x after time $t - t'$ [10, 12], we get the Cooperon from $\mathbf{F}$ by summing all final chiral states and averaging over initial chiral states: $C(x, t; x', t') = \frac{1}{2}\sum_{ij} F_{ij}(x, t; x', t')$. In matrix form, we get

$$C(x, t; x', t') = \mathrm{Tr}\left[\frac{1+\sigma_x}{2}\mathbf{F}(x, t; x', t')\right]. \tag{7}$$

Note that the main equation in (5) coincides in form with a Dirac equation for massive particles in (1 + 1) dimensions. Interestingly, it is already well known that the conventional Cooperon equation (1) coincides in form with the Schrödinger equation with imaginary time $t \longleftrightarrow -\mathrm{i}t$ for a particle with mass $m' \longleftrightarrow \hbar/2D$. To be more explicit, let us consider the following relativistic propagator G^{rel} for the Dirac equation for the particle with mass m' in (1 + 1) dimensions:

$$\left[\mathrm{i}\gamma^0\left(\frac{\partial}{\partial t} + \mathrm{i}\frac{m'c^2}{\hbar}\right) - \mathrm{i}c\gamma^1\left(\frac{\partial}{\partial x} - \frac{2\mathrm{i}e}{\hbar c}A\right) + \frac{m'c^2}{\hbar}\right]G^{rel} = \mathrm{i}\gamma^0\delta(x - x')\delta(t - t'), \tag{8}$$

where $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $g^{\mu\nu} = \mathrm{diag}(1, -1)$ for $\mu, \nu = 0, 1$. The term $\mathrm{i}\frac{m'c^2}{\hbar}$ within the first parentheses is included to compensate for the time-evolution factor $\exp(-\mathrm{i}\frac{m'c^2}{\hbar}t)$ due to the rest-mass energy which is not included in the Schrödinger equation. Let us perform the following transformations, while keeping the electron coupling constant $\frac{2\mathrm{i}e}{\hbar c}$ to the EM field:

$$t \to -\mathrm{i}t, \qquad c \to \mathrm{i}v_F, \qquad m' \to \hbar/2D; \tag{9}$$

then we get equation (5) using the Pauli spin matrices $\gamma^0 = \sigma_x$ and $\gamma^1 = \mathrm{i}\sigma_y$.

The physical origin of the transformation $c \to \mathrm{i}v_F$ in equation (9) is rendered clear by noting that the mean speed of a particle in a disordered conductor is limited by the Fermi velocity while the speed of a particle in the relativistic theory cannot exceed the speed of light. Instead of σ_x, chosen as γ^0 in the above, a general form of equation (7) can be used based on γ-matrices; $C(x, t; x', t') = \mathrm{Tr}[(\frac{1+\gamma^0}{2})\mathbf{F}(x, t; x', t')]$. The appearance of $(1 + \gamma^0)/2$ in front of $\mathbf{F}$ seems to be natural, because $(1 + \gamma^0)/2$ in relativistic mechanics is a projection operator, which projects out negative energy states in the rest reference frame [13].

Pursuing further the similarity between the present theory and relativistic theory, let us note how the chiral symmetry is broken in each theory. In relativistic quantum mechanics in (1 + 1) dimensions, the particle's mass breaks the chiral symmetry. (In other words, since the massive particle moves slower than c, there exist reference frames where a right-moving particle can be seen as a left-moving particle.) In mesoscopic quantum wires, the chirality is broken due to the presence of the electron–impurity scattering. These two different mechanisms, which break the chiral symmetry in each theory, are connected to each other as clearly manifested by the correspondence shown in equation (9); $m'c^2 \longrightarrow -\hbar v_F^2/2D = -\hbar/2\tau$.

In the absence of external fields, $\mathbf{F}$ is a translationally invariant quantity, which allows us to solve equation (5) using Fourier transformation: $\mathbf{F}(x, t; x', t') = (1/2\pi)^2 \int \mathrm{d}Q \int \mathrm{d}\omega\, \mathbf{F}(Q, \omega)\mathrm{e}^{\mathrm{i}Q(x-x')-\mathrm{i}\omega(t-t')}$. For $\tau_\phi \gg \tau$, $\mathbf{F}(Q, \omega)$ is given by

$$\mathbf{F}(Q, \omega) = \frac{1}{-\mathrm{i}\omega + DQ^2 - \omega^2\tau}\begin{pmatrix} 1/2 - \mathrm{i}\omega\tau - \mathrm{i}lQ & 1/2 \\ 1/2 & 1/2 - \mathrm{i}\omega\tau + \mathrm{i}lQ \end{pmatrix}. \tag{10}$$

The Cooperon in momentum space is written as

$$C(Q, \omega) = \mathrm{Tr}\,\frac{1+\sigma_x}{2}\mathbf{F}(Q, \omega) = \frac{1 - \mathrm{i}\omega\tau}{-\mathrm{i}\omega + DQ^2 - \omega^2\tau}. \tag{11}$$

This result coincides with the Cooperon obtained as the total sum of the Dyson series in the Green function approach without the approximations in equation (2) [14].

In the presence of a time-dependent EM field, $\boldsymbol{A} = A(t)\hat{\boldsymbol{x}}$, the 'Cooperon matrix' $\mathbf{F}$ explicitly depends on time t ($\mathbf{F}_t = \mathbf{F}_t(x, \eta; x', \eta')$), which is obtained by solving the following equation with a time-dependent field $A_t(\eta) = A(t - \eta/2) + A(t + \eta/2)$ [1]:

$$\left[\frac{\partial}{\partial\eta} + v_F\sigma_z\left(\frac{\partial}{\partial x} - \frac{\mathrm{i}e}{\hbar c}A_t(\eta)\right) + \frac{1-\sigma_x}{2\tau} + \frac{1}{\tau_\phi^*}\right]\mathbf{F}_t = \delta(x - x')\delta(\eta - \eta'). \tag{12}$$

Here, we include the phenomenological dephasing rate $1/\tau_\phi^*$, which originates from sources other than the external EM field. The weak-localization current $I_{WL}(t)$ [1] (the quantum correction to the classical ohmic current) is given by

$$\langle I_{WL}(t)\rangle = \frac{C_\beta e^2 D}{\hbar}\int_0^\infty \mathrm{d}\eta\, \mathrm{Tr}\left[\frac{1+\sigma_x}{2}\mathbf{F}_{t-\eta/2}(x, \eta; x, -\eta)\right]E(t - \eta), \tag{13}$$

where $\langle\cdots\rangle$ represents the disorder average and $E(t) = -\frac{1}{c}\frac{\partial A(t)}{\partial t}$ is the applied electric field. C_β is dictated by the Dyson symmetry class: $C_\beta = -4/\pi$ $(2/\pi)$ when the spin–orbit scattering is negligible (important) with the characteristic length $L_{so} \gg L$ ($L_{so} \ll L$) [1, 9].

Now, let us apply equations (12) and (13) to calculate electric currents induced by the EM field in mesoscopic rings. While the usual equilibrium persistent current is induced by a static magnetic flux $\phi = \bar{A}L$ only, the rectified direct current is a dynamical phenomenon originating from the time-dependent conductivity of the ring [5, 9]. Suppose the EM field, given by $A(t) = \bar{A} + a(t)$, is applied to the quantum ring with perimeter L, where $a(t) = \frac{1}{2}(a_\omega \mathrm{e}^{-\mathrm{i}\omega t} + \mathrm{c.c.})$ and $\bar{A}$ is time independent. An electric field $\mathcal{E}(t) = \frac{1}{2}(\mathcal{E}_\omega \mathrm{e}^{-\mathrm{i}\omega t} + \mathrm{c.c.})$ is induced along the ring, where $\mathcal{E}_\omega = \mathrm{i}\omega a_\omega/c$. The DC component of the electric current $I_0 = \overline{\langle I_{WL}(t)\rangle}$ is of interest, and it is obtained by averaging the disorder-averaged current $\langle I_{WL}(t)\rangle$ over time t.

Let us first investigate the case of weakly time-dependent field so that the associated magnetic flux ϕ_ω is much smaller than the unit flux quantum $\phi_0 = h/|e|c$:

$$\phi_\omega = |\mathcal{E}_\omega|Lc/\omega \ll \phi_0. \tag{14}$$

We calculate up to the first-order perturbation term of $a_{t-\eta/2}(\eta') = a(t - \eta/2 - \eta'/2) + a(t - \eta/2 + \eta'/2)$ in $\mathbf{F}_{t-\eta/2}$:

$$\mathbf{F}_{t-\eta/2}(x, \eta; x, -\eta) = \mathbf{F}^{(0)}_{t-\eta/2}(x, \eta; x, -\eta) + \int \mathrm{d}x' \int_{-\eta}^{\eta} \mathrm{d}\eta'\, \mathbf{F}^{(0)}_{t-\eta/2}(x, \eta; x', \eta') \times \left(v_F\frac{\mathrm{i}e}{\hbar c}\sigma_z a_{t-\eta/2}(\eta')\right)\mathbf{F}^{(0)}_{t-\eta/2}(x', \eta'; x, -\eta) + \cdots, \tag{15}$$

where $\mathbf{F}^{(0)}_{t-\eta/2}(x', \eta'; x, -\eta)$ denotes the $\mathbf{F}$-matrix in the absence of a time-dependent field: $a_\omega = 0$. After a long but straightforward calculation, we get the expression for the DC:

$$I_0 = C_\beta\frac{|e|}{\tau_D}\left(\frac{\phi_\omega}{\phi_0}\right)^2 \sum_{m=-\infty}^{\infty} \frac{4\pi^2(\omega\tau_D)^2 k_m}{[(k_m^2 - (\omega\tau_f)^2 + \tau_D/\tau_\phi^*)^2 + (\omega\tau_D)^2][k_m^2 + \tau_D/\tau_\phi^*]} \times \left(1 - \frac{\tau}{\tau_D}(k_m^2 - (\omega\tau_f)^2 + \tau_D/\tau_\phi^*)\right), \tag{16}$$

where $k_m = 2\pi(m + 2\phi/\phi_0)$ ($\phi = \bar{A}L$ is the static magnetic flux), $\tau_D = L^2/D$ is the diffusion time, and $\tau_f = L/v_F$ is a ballistic timescale. By neglecting ballistic parameters in equation (16)—i.e., $\tau/\tau_D \to 0$ and $\omega\tau_f = \omega\tau_D\sqrt{\tau/\tau_D} \to 0$—we recover the earlier result for I_0 given by Kravtsov and Yudson [5]. Compared with the results for the diffusive limit [5], we basically encounter new parameters τ/τ_D on considering ballistic effects.

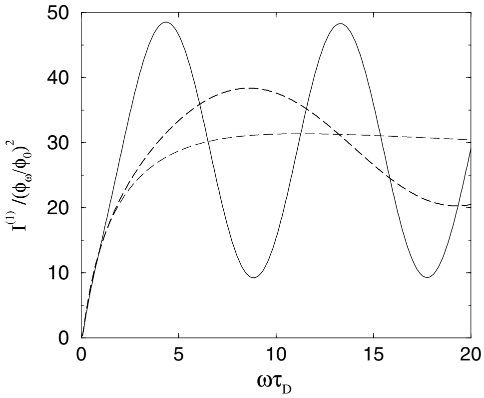

Figure 1. The amplitude $I^{(1)}$ of the first harmonic of $I_0(\phi)$ in units of $C_\beta|e|/\tau_D$ for a weakly time-dependent field $\phi_\omega \ll \phi_0$ and different 'ballisticities' $\tau/\tau_D = 0.5$ (thick solid curve), $\tau/\tau_D = 0.1$ (thick dashed curve), and $\tau/\tau_D = 0$ (dashed curve [5]). τ_ϕ^* was chosen to be $10\tau_D$ for all cases.

Since I_0 is periodic with a period $\phi_0/2$, the Fourier components $I^{(n)}$ of I_0 are often the quantities under study;

$$I_0(\phi) = C_\beta \frac{|e|}{\tau_D} \sum_n I^{(n)} \sin\left(4\pi n \frac{\phi}{\phi_0}\right). \tag{17}$$

In figure 1, we plot the amplitude of the first harmonic $I^{(1)}$ of $I_0(\phi)$ using equation (16). In contrast with the current for the diffusive limit [5], i.e. $\tau/\tau_D = 0$, $I^{(1)}$ for finite τ/τ_D shows oscillation behaviour. A new timescale $\tau_f = L/v_f$ appears associated with the oscillation period $\Delta\omega = 2\pi/\tau_f$. Note that when we take into account ballistic effects (i.e., $\tau/\tau_D \neq 0$), we cannot neglect $\omega\tau_f$ $(=\omega\tau_D\sqrt{\tau/\tau_D})$ in the denominator of equation (16), which gives oscillating behaviour in figure 1. Intuitively, this oscillation is due to the fact that the time period of periodic orbits along the ring matches with that of the applied external field.

Now, let us look at a different regime where the disorder potential is very weak but the applied field is arbitrarily strong:

$$\frac{1}{\tau} \ll \omega \qquad \text{and} \qquad \frac{1}{\tau_f}. \tag{18}$$

For this case, we use perturbation of the electron–impurity scattering term σ_x/τ with the parameter $1/\omega\tau \ll 1$. The leading terms are written as

$$I^{(n)} \approx \mathcal{F}_n\left(\pi\frac{\phi_\omega}{\phi_0}, \omega\tau_f\right) e^{-n\tau_f/\tau} \times \left[\sin(n\omega\tau_f/4) + \frac{1}{\omega\tau}\left(\frac{\cos(n\omega\tau_f/4) + (2/\omega\tau)\sin(n\omega\tau_f/4)}{1 + (2/\omega\tau)^2}\right) + \cdots\right], \tag{19}$$

where

$$\mathcal{F}_n(x, y) = xyJ_1(16x\sin(ny/4)/y). \tag{20}$$

Here J_1 is the Bessel function of order 1.

As shown in figure 2, the first harmonic $I^{(1)}$ of the current may show sign reversal when the applied field is not too weak. When the magnetic flux ϕ_ω associated with the time-dependent field is larger than half the flux quantum, $\phi_0/2$, $I^{(1)}$ is in a regime of negative sign depending

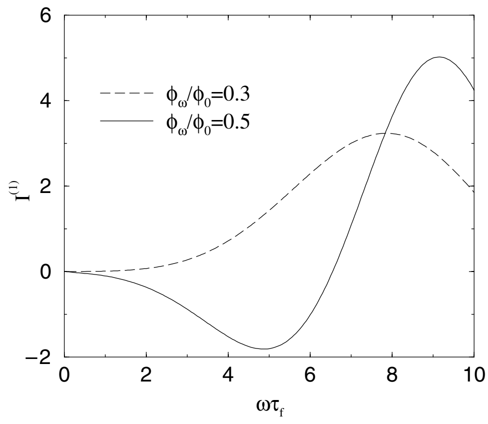

Figure 2. The amplitude $I^{(1)}$ of the first harmonic of $I_0(\phi)$ in units of $C_\beta|e|/\tau_D$ using the high-frequency approximation $1/\tau \ll \omega, 1/\tau_f$. $\tau_f/\tau = 0.1$ was chosen as a specific case.

on the applied frequency. Interestingly, this is also the condition for the applied field being able to cause dephasing of electrons efficiently.

The experimental configuration in [4] seems to be promising for the observation of the ballistic effects that we have discussed here. However, compared with metals, GaAs samples are more promising as regards showing ballistic effects—where the mean free path is usually the order of micrometres. Furthermore, well-defined amplitude and frequency are both necessary for comparisons. In the case of the GaAs samples, an applied field of frequency ω of the order of several terahertz may clearly show the ballistic effects that we discussed.

In conclusion, we have shown that the mesoscopic electron transport in disordered quantum wires can be described by a generalized Cooperon equation which coincides in form with the Dirac equation for massive fermions in a (1 + 1)-dimensional system. Ballistic effects in a disordered wire are equivalent to the relativistic effects in clean one-dimensional systems. On the basis of the new Cooperon equation, electric currents in mesoscopic rings induced by oscillating magnetic fields are calculated. It is predicted that, as a ballistic effect, the DC component of the induced electric currents will show oscillating behaviour in the domain of external-field frequency. Furthermore, in the high-frequency regime, the sign of the induced current can lead to either diamagnetism or paramagnetism, depending on the strength and the frequency of the field.

Acknowledgments

The author would like to thank C Kim, P Mohanty, L I Glazman, B I Halperin, D Kim, M Das, F Green, Y D Park, C Lee, M Y Choi, and T Yamamoto for stimulating discussions.

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while for a comprehensive study of the ballistic effects in two dimensions, we refer the reader to, e.g.,
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